Donsker's Invariance Principle is a key result in probability theory and functional analysis that establishes a functional analog to the Central Limit Theorem (CLT). While the traditional CLT deals with the convergence of sums of random variables, Donsker's Invariance Principle extends this to the convergence of certain functionals of random processes. The theorem is particularly important in the study of stochastic processes and empirical processes.

**Statement of Donsker's Invariance Principle:**

Let {��(�)}{*Xn*​(*t*)} be a sequence of independent and identically distributed (i.i.d.) random processes with mean zero and variance one. Define the empirical process ��(�)*Gn*​(*t*) as:

��(�)=1�∑�=1�[��(�)−�(��(�))]*Gn*​(*t*)=*n*​1​∑*i*=1*n*​[*Xi*​(*t*)−E(*Xi*​(*t*))]

If {��(�)}{*Xn*​(*t*)} satisfies some mild conditions, then the process ��(�)*Gn*​(*t*) converges in distribution, as �*n* approaches infinity, to a standard Brownian motion �(�)*B*(*t*).

**Proof:**

The proof of Donsker's Invariance Principle often involves functional analysis, probability theory, and characteristic function methods. A key step is to show that the finite-dimensional distributions of the empirical process converge to those of a Brownian bridge, and then leveraging the Skorokhod representation theorem to establish the weak convergence.

In this simulation, we generate five i.i.d. random processes and compute the corresponding empirical processes. The plot demonstrates the convergence of these empirical processes to a Brownian bridge, as indicated by the characteristic random-walk-like behavior.